Towards concurrent NetKAT

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University College London

CReNKAT kick-off workshop, 12/03/2019

Joint work with Paul Brunet, Bas Luttik, Jurriaan Rot, Alexandra Silva, Jana Wagemaker, Fabio Zanasi.

Introduction

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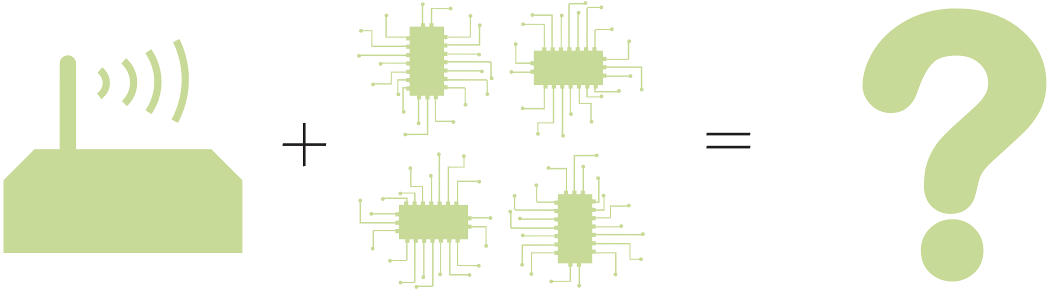
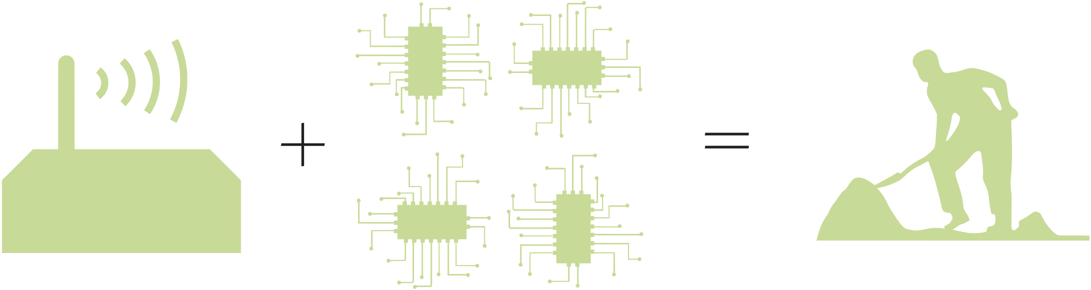
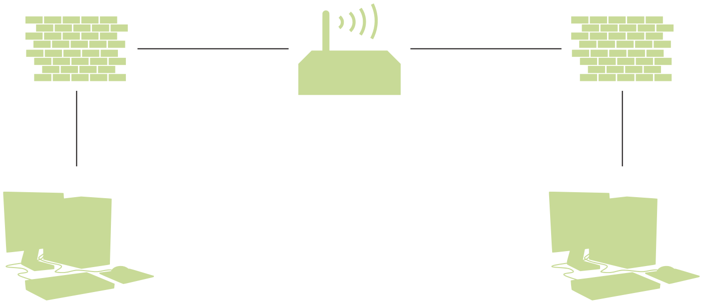


Introduction



What do you mean, concurrency?



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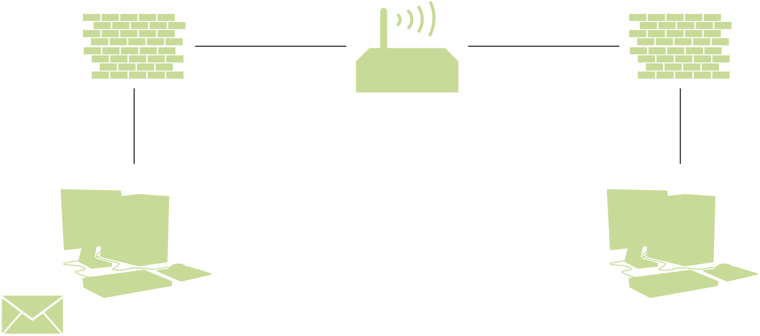

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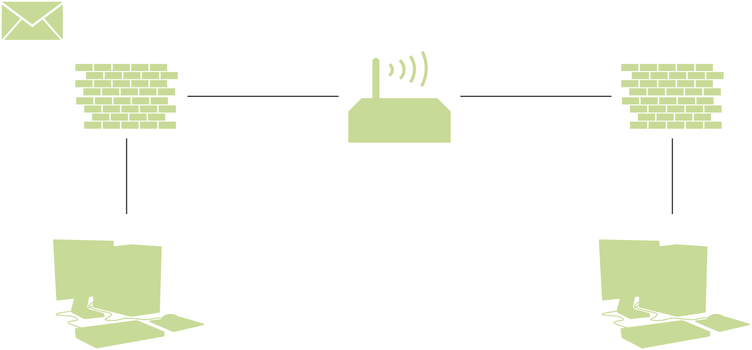

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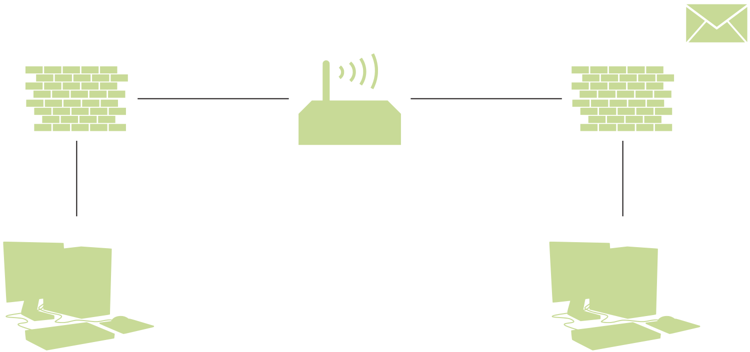

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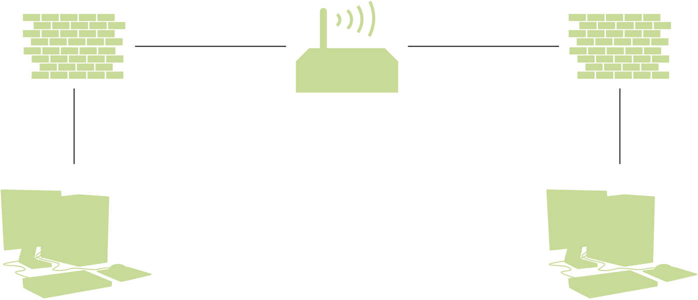

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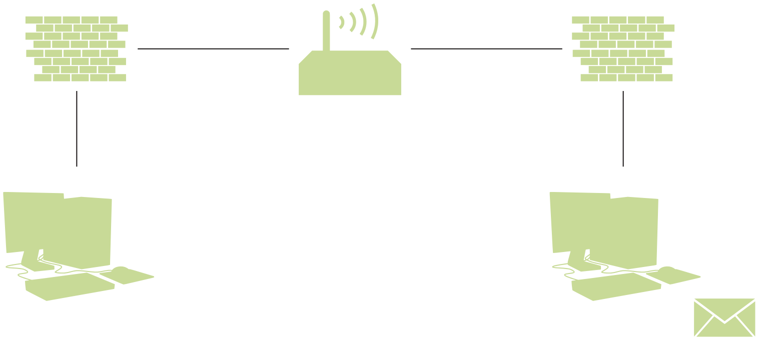

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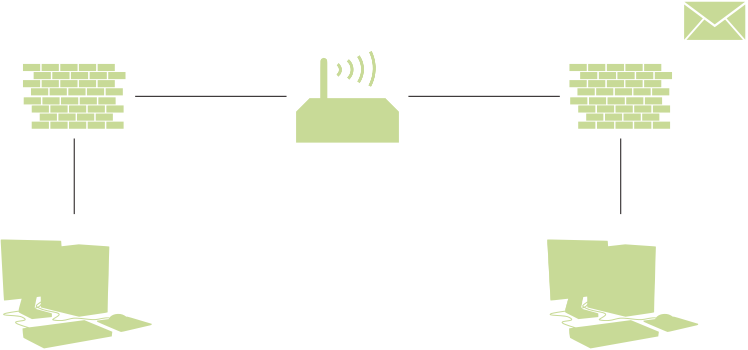

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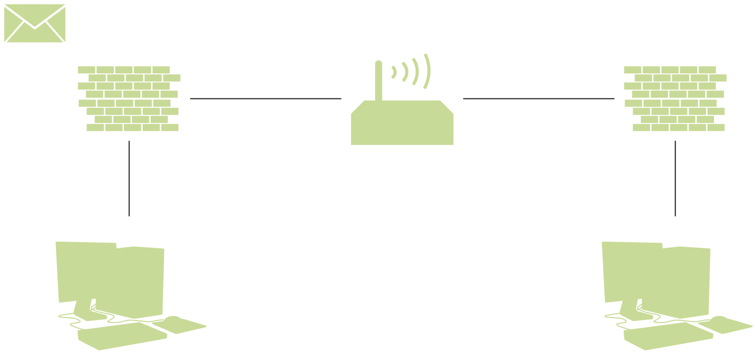

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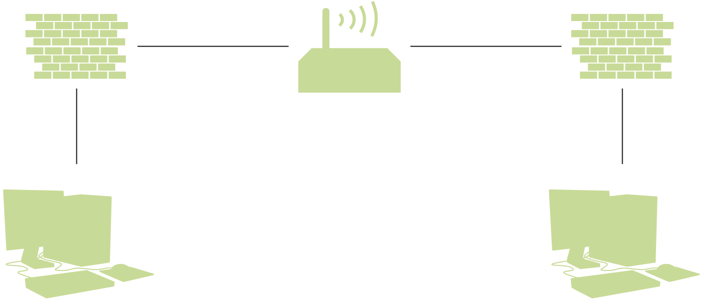

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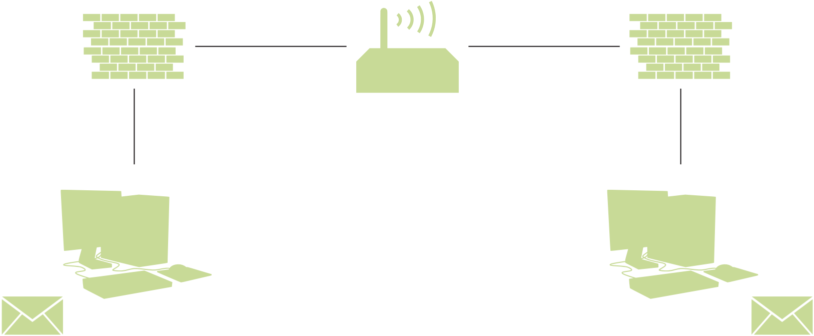

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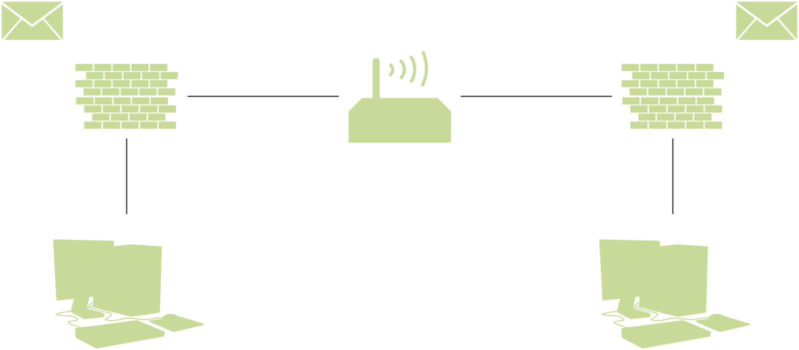

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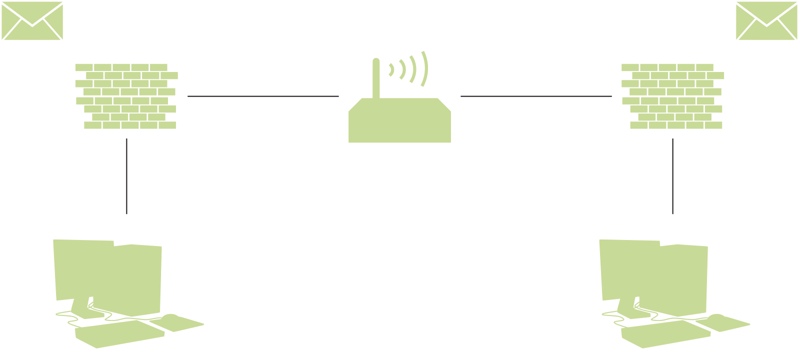

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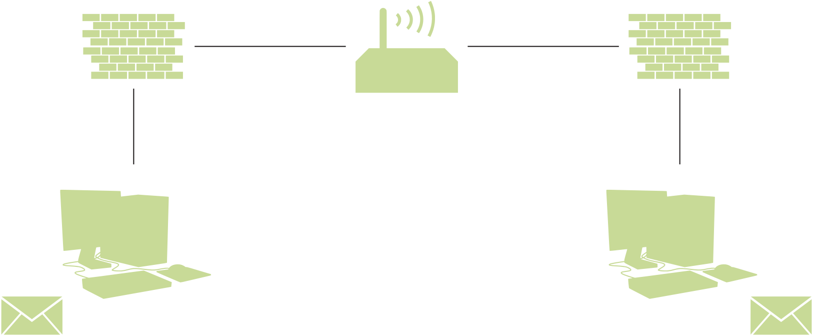

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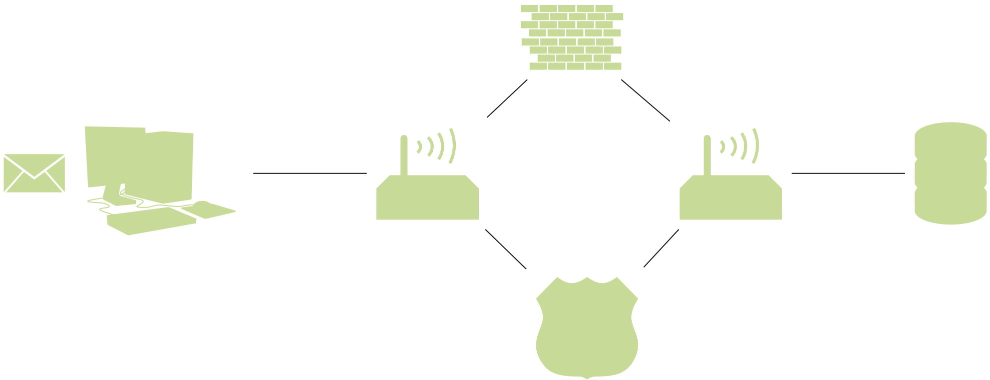

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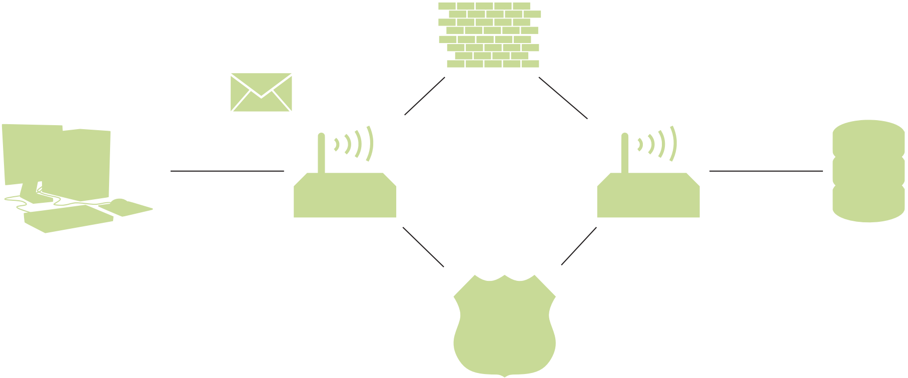

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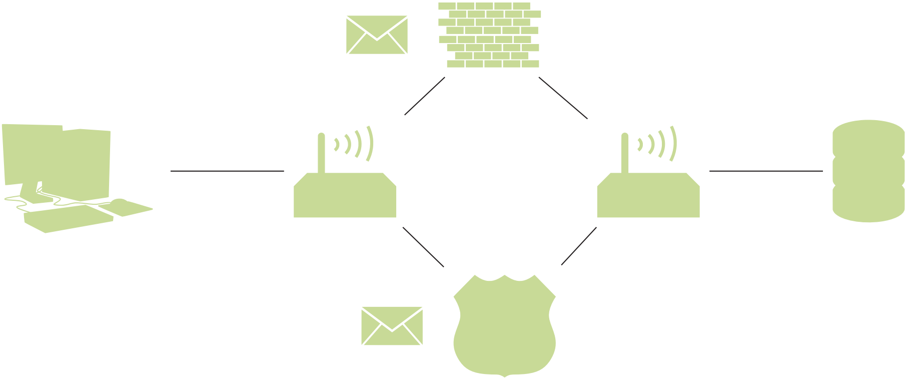

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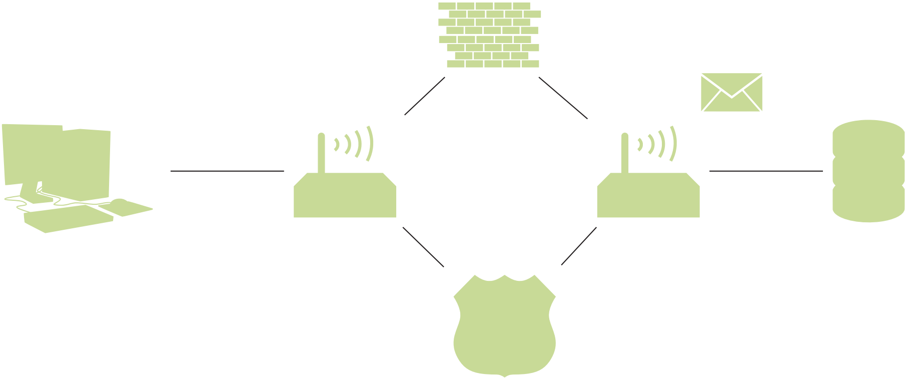

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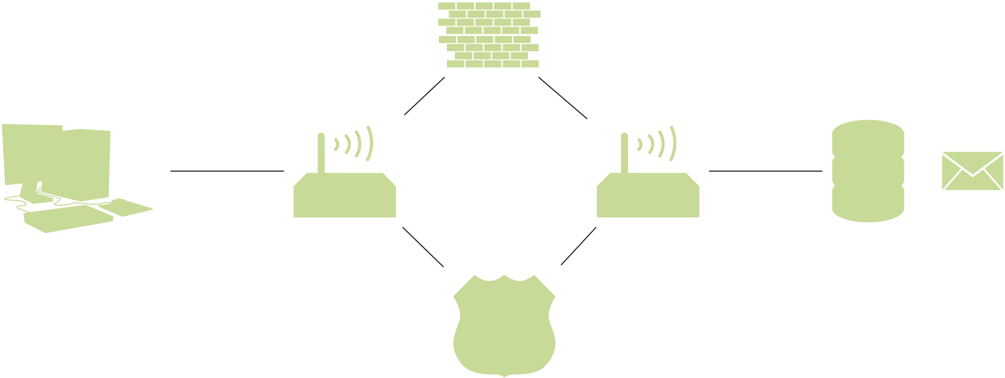

NetKAT plus parallel composition

$$e, f ::= 0 \mid 1 \mid f \leftarrow v \mid f = v \mid e + f \mid e \cdot f \mid e^*$$

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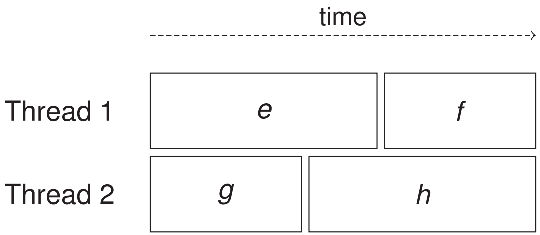
$$e \parallel f \equiv f \parallel e \qquad e \parallel (f \parallel g) \equiv (e \parallel f) \parallel g \qquad e \parallel 0 \equiv 0 \qquad e \parallel 1 \equiv e$$

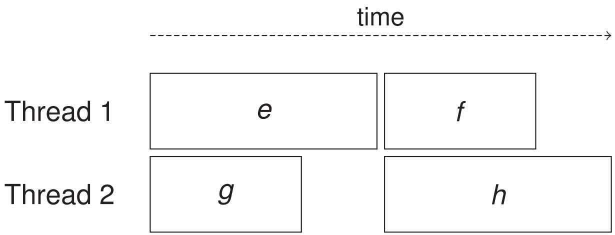
NetKAT plus parallel composition

$$(e \cdot f) \parallel (g \cdot h)$$



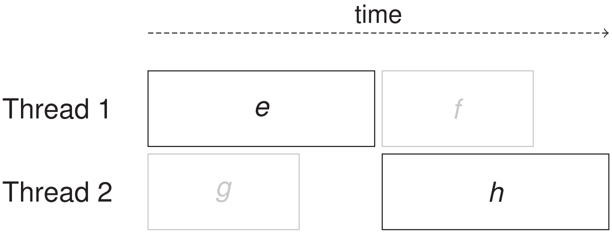
NetKAT plus parallel composition

$$(e \parallel g)(f \parallel h)$$



NetKAT plus parallel composition

$$(e \parallel 1)(1 \parallel h)$$

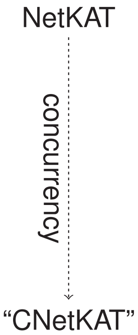


NetKAT plus parallel composition

Why not do total interleaving?

- Requires synchronizing packet state across nodes.
- Individual copies may be modified along the way.

NetKAT plus parallel composition



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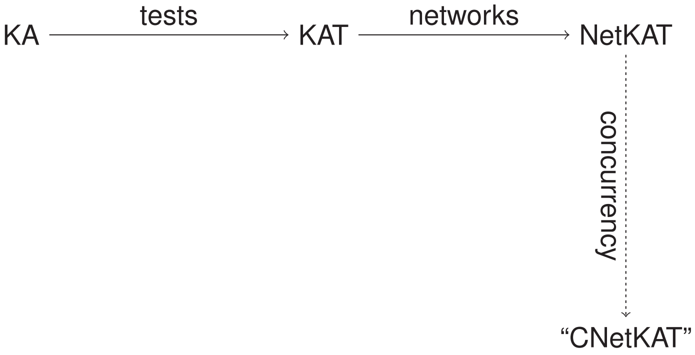

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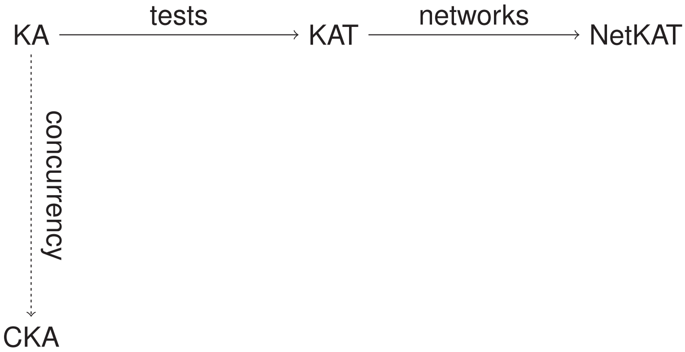
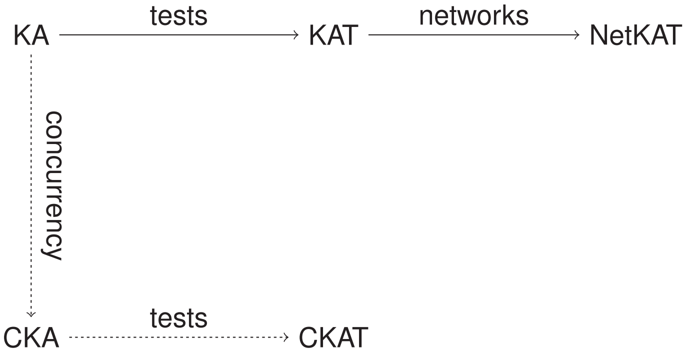


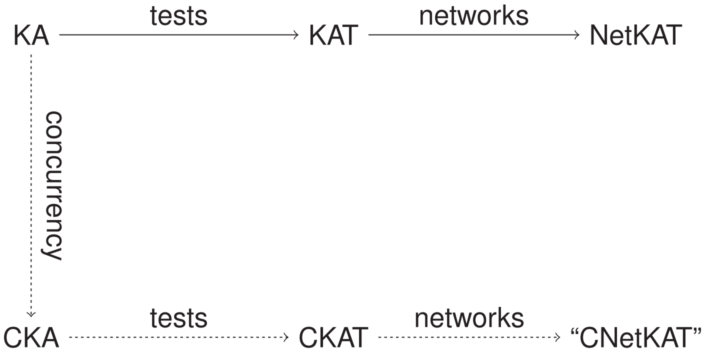
See e.g. [HMS+]

NetKAT plus parallel composition



See e.g. [JM]

NetKAT plus parallel composition

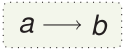


Concurrent Kleene Algebra

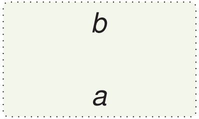
$$a \cdot b \quad \approx \quad \boxed{a \longrightarrow b}$$

Concurrent Kleene Algebra

$a \cdot b \approx$

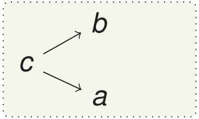


$a \parallel b \approx$



$$a \cdot b \approx a \longrightarrow b$$

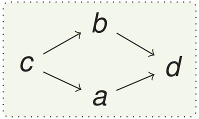
$$c \cdot (a \parallel b) \approx$$



Concurrent Kleene Algebra

$$a \cdot b \quad \approx \quad a \longrightarrow b$$

$$c \cdot (a \parallel b) \cdot d \quad \approx$$



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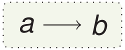
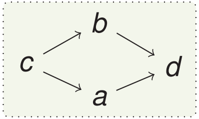
$a \cdot b \quad \approx$ 

$c \cdot (a \parallel b) \cdot d \quad \approx$ 

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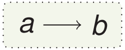
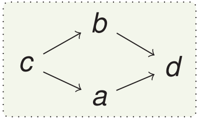
$a \cdot b \approx$ 

$c \cdot (a \parallel b) \cdot d \approx$ 

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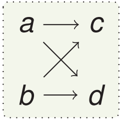

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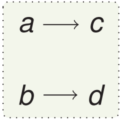

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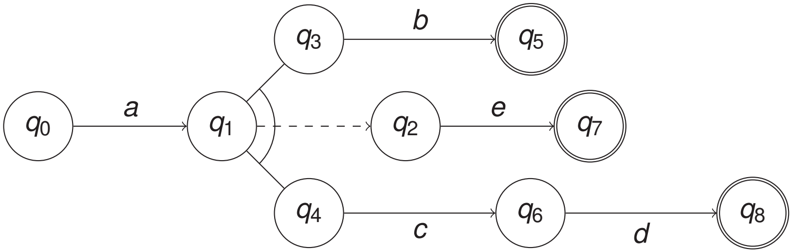
pairwise

Theorem [BPS]

Given terms e and f, it is decidable whether $[\![e]\!] = [\![f]\!]$.

Theorem [KBS+]

Given terms e and f, we have that $e \equiv f$ *if and only if* $[\![e]\!] = [\![f]\!]$.



q_0
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q_6
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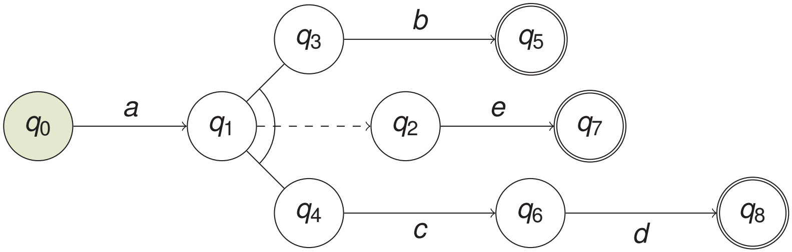
q_8

Concurrent Kleene Algebra



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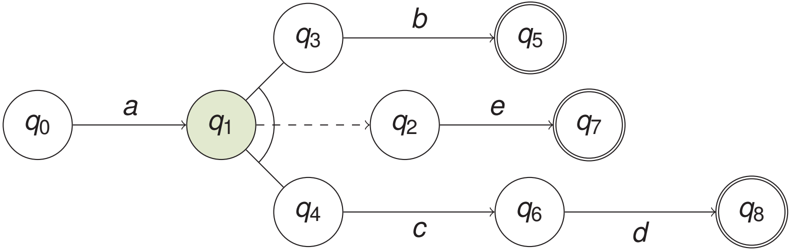

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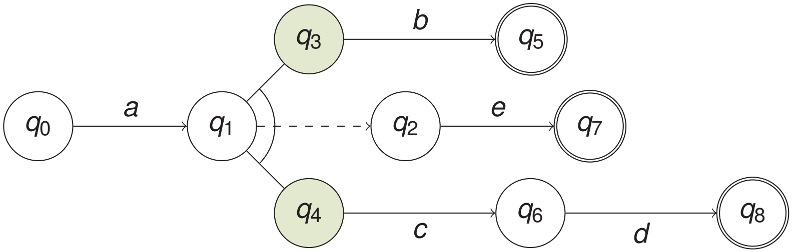

a

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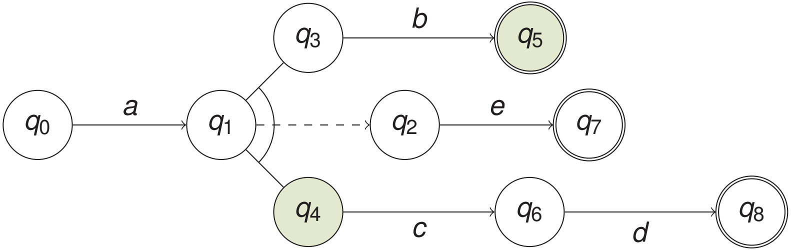

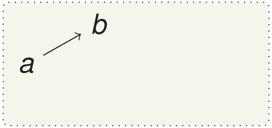

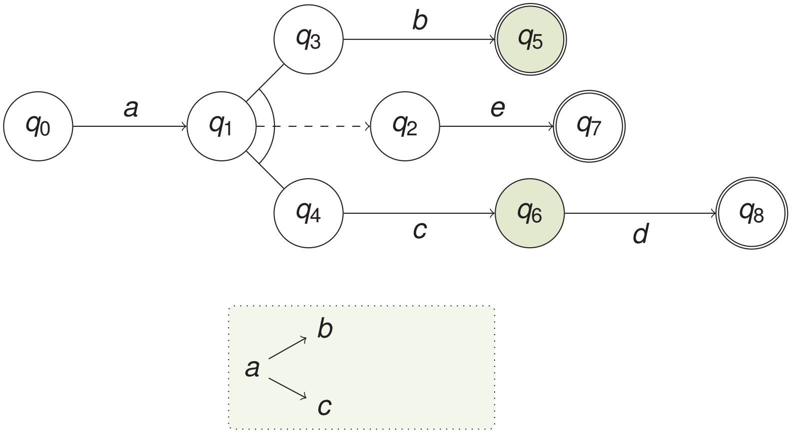

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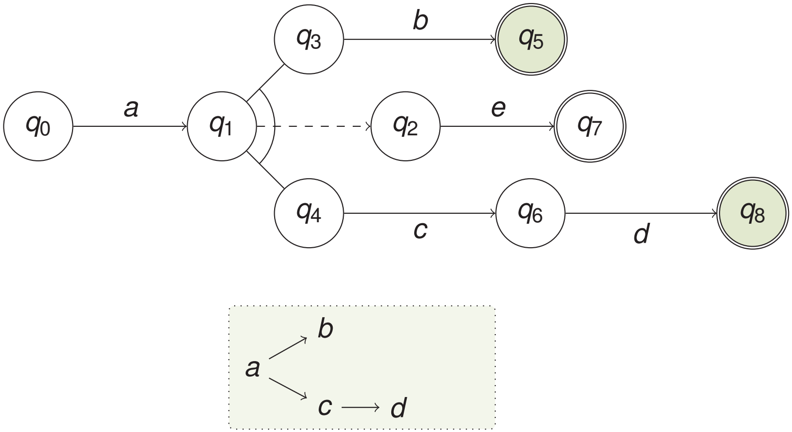

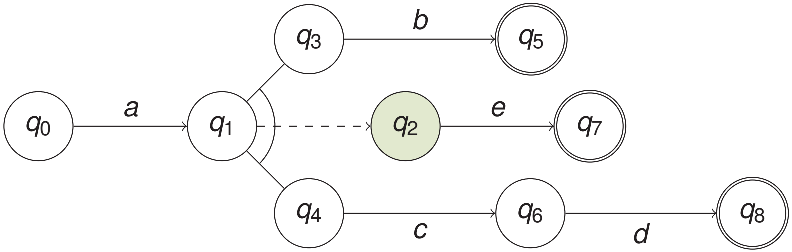

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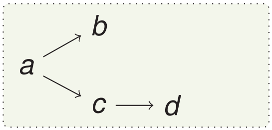

a
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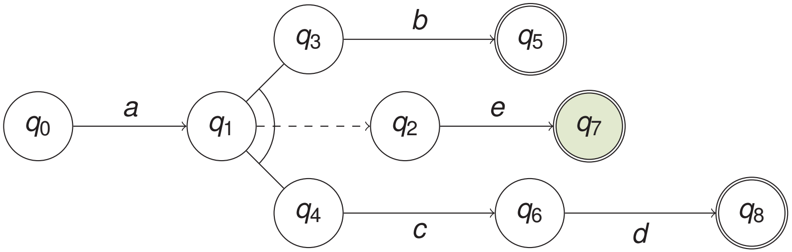

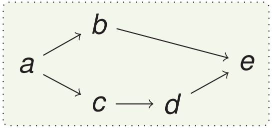

Concurrent Kleene Algebra

Theorem [KBS+; KBL+]

Let L be a pomset language. The following are equivalent:

- *L is recognized by a series-rational expression e.*
- *L is recognized by a fork-acyclic and well-structured pomset automaton.*

Concurrent Kleene Algebra

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Let L be a pomset language. The following are equivalent:

- *L is recognized by a series-rational expression e.*
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Theorem [KBL+]

Language equivalence of fork-acyclic and well-structured pomset automata is decidable.

Concurrent Kleene Algebra. . . with tests?

$$e, f ::= \quad a \in \Sigma \mid e + f \mid e \cdot f \mid e^* \mid e \parallel f$$

Concurrent Kleene Algebra... with tests?

$$p, q ::= 0 \mid 1 \mid o \in \mathcal{O} \mid p \vee q \mid p \wedge q \mid \overline{p}$$

$$e, f ::= p \mid a \in \Sigma \mid e + f \mid e \cdot f \mid e^* \mid e \parallel f$$

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$$p + q \equiv p \vee q \qquad p \wedge q \equiv p \cdot q \qquad p \wedge \overline{p} \equiv 0$$

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$$p \cdot e \cdot \overline{p}$$

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$$p \cdot e \cdot \overline{p} \leqq (p \cdot \overline{p}) \parallel e$$

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$$p \cdot e \cdot \overline{p} \leqq (p \cdot \overline{p}) \parallel e \equiv (p \wedge \overline{p}) \parallel e \equiv 0 \parallel e$$

Concurrent Kleene Algebra. . . with tests?

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Theorem preprint: [KBR⁺]

There is a language semantics $[\![-]\!]$ such that

1. $[\![e]\!] = [\![f]\!]$ *if and only if* $e \equiv f$, *and*
2. *it is decidable whether* $[\![e]\!] = [\![f]\!]$.

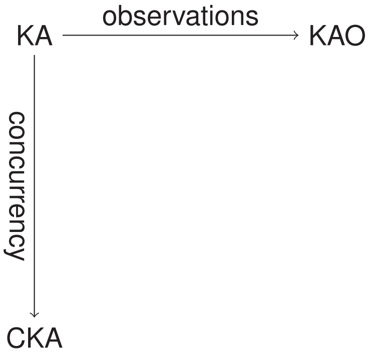
Concurrent Kleene Algebra. . . with tests?

KA

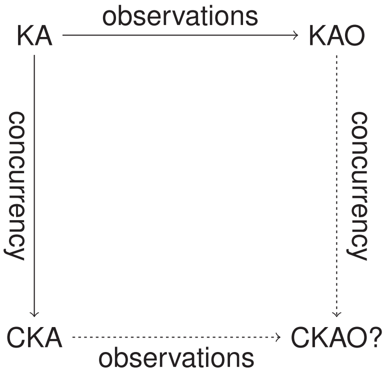
Concurrent Kleene Algebra. . . with tests?

$$\text{KA} \xrightarrow{\text{observations}} \text{KAO}$$

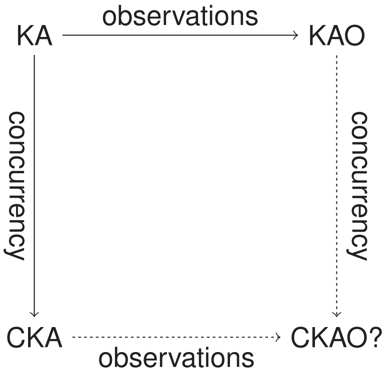
Concurrent Kleene Algebra. . . with tests?



Concurrent Kleene Algebra. . . with tests?



Concurrent Kleene Algebra. . . with tests?



$$[p \cdot q] \equiv [p \wedge q]$$

Multicasting or nondeterminism?

Hurdles on the horizon

Multicasting or nondeterminism?

$$e + f \equiv f \iff e \leq f$$

Hurdles on the horizon

True concurrency versus sequential consistency?

True concurrency versus sequential consistency?

$$(f \leftarrow v) \cdot (f' \leftarrow v') \equiv (f' \leftarrow v') \cdot (f \leftarrow v)$$

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